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Asteroid Close Encounters with Earth: Risk Assessment

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Abstract

When an asteroid with an orbit close to the Earth's is observed only over a short arc, there is no way to accurately predict the future close approaches. The first approach could change the orbital period in such a way that the asteroid is injected into a resonance, allowing an even closer approach a few years later. By using both a qualitative theory for these resonant returns, and a systematic numerical investigation of alternate orbital solutions, it is possible to identify future encounters and to assess their impact risk. This allows one to monitor the impact risk by all known near-Earth asteroids.

1 The problem

The problem is to assess the risk of an impact with the Earth by a potentially hazardous object, either an asteroid or a comet, and in the worst case to evaluate the feasibility and risk of a possible deflection initiative. We address here the risk from asteroids which have already been discovered, and for which an orbit is available; the risk from yet undiscovered objects is a separate issue.

If an asteroid has already been observed over not too short of an arc, then an estimate of the orbital elements X is available; the *nominal solution* is the one corresponding to a minimum of the sum of squares of the (suitably weighted) observation residuals. Around the nominal solution X^* there is a *confidence region* Z_X such that the residuals from a solution $X \in Z_X$ are not significantly higher than the ones for the nominal solution; what "significantly higher" means is controlled by a confidence parameter σ [1].

The orbits corresponding to the initial conditions $X \in Z_X$ propagate with time, forming a tube in phase space (also in physical, 3-D space). If this space-time tube intersects the other tube formed by the Earth surface, then a collision is possible; by possible we really mean compatible with the available observations of the asteroid, in that the residuals RMS is not increased beyond some threshold of acceptability.

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There are two connected but conceptually different problems: to detect the *possibility* of an impact, and to compute its *probability.* The latter can be computed only if some probability density function is given for the orbital elements X, which is in turn a consequence of an assumed probability density for the observation errors. These probability densities are indeed available if the gaussian theory is applicable, but that is seldom the case with sparse astrometric observations plagued by systematic errors, as is typical for asteroid observations.

In this paper we address systematically the issue of detecting impact possibilities, and give only order of magnitude estimates of impact probability, based upon approximate computations of the size of the subset of Z_X for which collisions occur. In Sec. 2 we compute the intersection of the tube of possible orbits with the *target plane* of some Earth encounter (around a given time). In Sec. 3 we describe the situation in which the alternate orbits (all belonging to Z_X) are so widely spread out, that they can complete a different number of revolutions around the Sun, and thus some of them can approach the Earth as close as the minimum distance between the osculating ellipses; then deflection into a safe orbit is a nontrivial problem. In Sec. 4 we describe a complete procedure to detect possible close approaches, evaluate the minimum possible distance, and estimate the impact probability, if any. We also provide some examples, and discuss a case in which one such impact possibility belongs to an asteroid which is lost.

2 Target plane analysis

Let t^* be a time, much later than the epoch of observations, when the nominal orbit with initial conditions X^* has a minimum in the distance from the Earth. The plane through the Earth and orthogonal to the geocentric velocity of the asteroid at that time is the *Modified Target Plane* (MTP). The orbits X with initial conditions in the neighborhood of X^* will also intersect the MTP at a time $t(X) \simeq t^*$, at a point $Y = F(X)$; F is a differentiable function [16].

Conceptually, it is possible to decide if an impact can occur by computing $F(Z_X)$ and by checking if this *MTP confidence region* intersects the cross section of the Earth. In practice, only a finite number of orbits can be computed, and to sample a six-dimensional region such as Z_X a very large number of initial conditions would have to be selected. To solve this computational problem a sequence of approximations is available [1]. The linear method uses the linearization of the map F, represented by the matrix DF of partial derivatives, and can only describe the MTP confidence region as a *confidence ellipse.* The semilinear method uses the *semilinear confidence boundary* [16], that is the image under F of the same ellipse in Z_X that would give the confidence ellipse under DF. Fully nonlinear methods, including Monte Carlo sampling of Z_X according to the given probability density, are available, but the computational cost is high.

When the close approach is far from the observed arc the MTP intersection has a very elongated shape. This results essentially from the propagation of the semimajor axis uncertainty into an increasing uncertainty of the mean anomaly as time goes by. The main example we are going to discuss in this paper is the asteroid 1999 AN_{10}, which was initially observed only between January 13 and February 20, 1999, then was temporarily unobservable, being at a low elongation from the Sun. The nominal orbit computed with these early observations has a close approach near the ascending node in August 2027 at

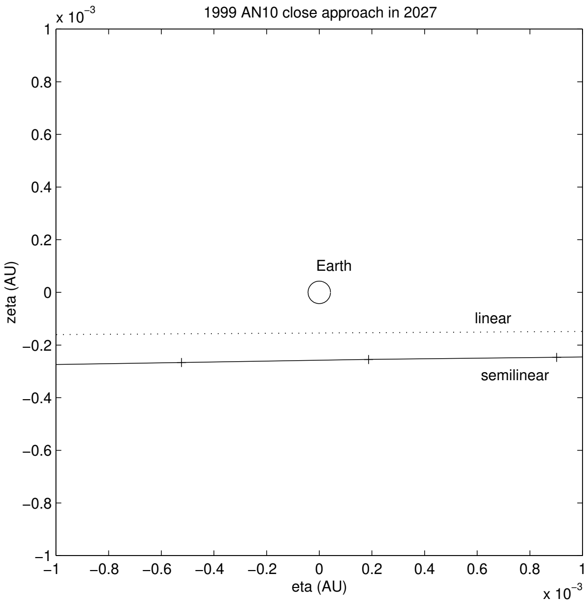

Figure 1: The close approach of August 2027 and its uncertainty as projected on the Modified Target Plane, using only the observations until February 20, 1999. The prediction according to the linear theory would allow for an approach closer than the one allowed by the more accurate semi-linear theory. In any case, the Earth (shown to scale) is not at risk.

a distance from Earth's center of 3.8 million km. With only the observations over this short arc, the orbit has a significant uncertainty in the semimajor axis, to the extent that the time at which the orbit would intersect the ecliptic plane in the summer of 2027 is uncertain by ± 12 days. The confidence region (for $\sigma = 3$) is more than 60 million km long, but it is only 2 600 km wide. This very narrow strip can pass significantly closer to the Earth than the nominal approach, the minimum distance within the confidence region is only $37,000$ km (from the center of the Earth). Under these conditions it is not surprising that the linear approximation gives a poor estimate of the minimum possible close approach distance; Fig. 1 shows the linear and semilinear confidence boundaries near the Earth, and quite far from the nominal point $F(X^*)$. In this case, however, the semilinear methods gives a reliable answer, which has been confirmed by Monte Carlo simulations [3].

Thus for each close approach found along the nominal solution, target plane analysis should be performed to establish the minimum possible approach distance; the use of the linear approximation is legitimate when the closest approach distance turns out to be not much less than the nominal one, as in the case of 1997 XF_{11} [2, 16]. The semilinear method should be used in dubious and more difficult cases, such as 1999 AN_{10}.

The main limitation of the target plane analysis such as the one above is the following: a close approach of the nominal solution is needed to begin with. But, what is a close approach? The distance d up to which an encounter is considered "close" is quite arbitrary; in practice, it is difficult to use $d > 0.15$ AU, otherwise there could be simultaneous close approaches to the Earth and to Venus; we use $d = 0.1$ AU. As the 1999 AN_{10} encounter in 2027 shows, the uncertainty of the orbit can be such that the occurrence of a "close" encounter needs to be discussed in probabilistic terms. In that case the longest semiaxis of the MTP confidence ellipse (corresponding to $\sigma = 1$) was 0.07 AU; the ratio $0.07\,\mathrm{AU}/1\,\sigma$ is the *MTP stretching* [14], denoted by Γ_{MTP}. If the stretching becomes larger, e.g., several AU, then the close approach occurs only for a small portion of the confidence region Z_X. This implies that monitoring the close approaches along the nominal solution is not enough; impact solutions can occur for initial conditions well inside Z_X, but at a time when the nominal position is very far from the Earth.

3 Returns to close approaches

The idea of resonant returns has been in use for a long time in astrodynamics, although the name was introduced in [14]. In the late 1960's G. Colombo proposed to use the deflection resulting from the first encounter with Mercury to inject the Mariner 10 spacecraft into a resonance with the same planet; his proposal was actually implemented in the 1970's, thus Mariner 10 visited Mercury three times without additional cost in fuel. The technique is now called multiple gravity assist and is considered as standard in the mission analysis of space probes going either to the outer solar system or to Mercury. The same idea had been used long ago to discuss the dynamics of comet Lexell [9], [10], which had been inserted by a first encounter with Jupiter into a 2/1 Jovian resonance. Only recently has it been proposed to apply the same concept to discuss the possibility of future close approaches of asteroids to the Earth [11].

3.1 Multiple solutions

One intuitive way of expressing the uncertainty of the orbit of an asteroid is to use a finite probabilistic model. We can describe the confidence region Z_X by thinking of a swarm of *virtual asteroids*, with slightly different orbits all compatible with the observations. The reality of the asteroid is shared among the virtual ones, in the sense that only one of them is real, but we do not know which one. To make things simple, we can think that all the virtual objects are equally likely to be the real one; a more complicated probability law could be used, changing the exact values of the estimated probabilities but not the qualitative picture. Since the confidence region contains a continuum of orbits, each virtual asteroid is in turn representative of a small region, that is its orbit is also uncertain, but to a much smaller degree [1]. Note that the so called "nominal" orbit is just one of the virtual asteroids, without any special significance.

The confidence region can be sampled by virtual asteroids in different ways in accordance with some given probability density in the X space, e.g., the Monte Carlo methods use random sampling [4, 19]. As time goes by, however, it matters less and less how the virtual asteroids are distributed: as a result of the differences in their semimajor axes, hence periods, they gradually separate along track, until they become like a string of pearls, with comparatively small cross track separations. This implies that the most efficient way to sample the tube of orbits, for a long span of time after the observed arc, is to sample the confidence region Z_X along a one-dimensional subset, provided the semimajor axis changes along it. Thus we have adopted the method of *multiple solutions* from Ref. [13, Sec. 5],

[1] This is a non-mathematical description, which could be transformed in a rigorous definition by using the mathematical notions of *probability density function* and of *partition of unity.*

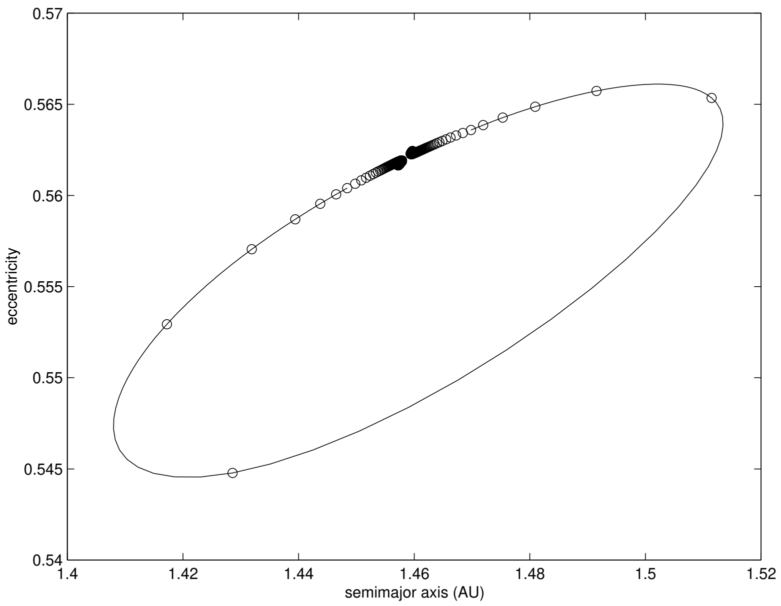

Figure 2: The orbital elements (a, e) of 1999 AN_{10} after the 2027 close approach; the circles are 1001 multiple solutions evenly spaced for $\sigma \leq 3$, the continuous line has been obtained with denser sampling where the encounter is deep.

which gives uniformly spaced (in σ units) solutions along the weakness line, that is the curve in Z_X always tangent to the eigenvector associated with the largest eigenvalue of the local covariance matrix.

When an asteroid undergoes a close approach in the future, several decades after the last available observation, the confidence region on the MTP is thin (typically with a width much less than the diameter of the Earth) and very long. This is due to the along track separation, which results in the asteroid being either early or late at its date with the Earth at the closest point. Thus the intercepts on the MTP of the multiple solutions computed as above, with points close to the long axis, are an efficient way of sampling the MTP confidence region.

3.2 Resonant returns

We can formulate the theory of resonant returns as follows [14]. If we assume that an asteroid undergoes an encounter several decades after the last available observation, the confidence region on the MTP is very long, with a stretching that can be several AU. Then only some of the virtual asteroids can undergo a close approach, and only very few will have an approach at a distance close to the minimum possible. The multiple solutions undergo different degrees of perturbations, as a result of the close approach, ranging from an essentially unperturbed orbit when the approach is very shallow to a large, almost impulsive change of the orbital elements when the approach is closest.

The post-encounter elements of a string of virtual asteroids, obtained as described in the previous subsection, describe a curve in the orbital elements space, e.g., in the (a, e)-plane; an idea of the shape of such curves can be obtained by using Öpik's approximation, which

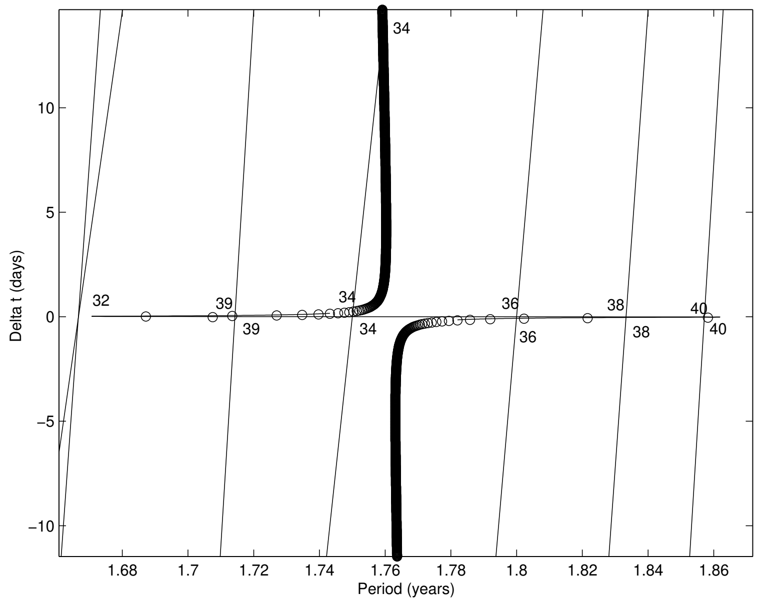

Figure 3: Resonant returns of 1999 AN_{10} after the 2027 encounter, taking place until August 2040. The resonant returns are the intersections between the straight lines and the string of multiple solutions, all labeled with the year (after 2000) in which the return takes place.

is essentially a piecewise two body solution, and is reasonably accurate for encounters with a high relative velocity [5]. It turns out that these curves are almost closed, that is they go back to nearly the unperturbed values on both extremes of the string of multiple solutions where the encounter is shallow, as in Fig. 2.

The post-encounter orbital periods range between a minimum P_{min} and a maximum P_{max} [22]. Every rational number in the interval between P_{min} and P_{max} corresponds to a resonant return; if the period $P = h/k$ years with h, k integers, then after h years the asteroid has completed k orbits, the Earth has completed h orbits, and both return to nearly the same position as they were at the previous encounter. Because the curve in the (a, e)-plane is nearly closed , the lines $P = h/k$, which are of the form $a = const$, in general cross it an even number of times.

The above theory is still too simple, however, because two further arguments have to be taken into account. The first one is that the amount of time by which the first encounter has been missed needs to be recovered to make the second encounter a close approach. If Δt is the amount of time by which the asteroid is early for the encounter with the Earth, the condition to be satisfied for a resonant return at the minimum distance is

$$h + \Delta t = k\, P \ ,$$

where Δt and P are in years. Thus the resonant returns are described in the $(P, \Delta t)$ plane by lines which are somewhat slanted with respect to the $P = h/k$ lines. As an example, let us take the 2027 encounter of 1999 AN_{10}. Figure 3 depicts these resonant lines for $h \leq 13$, that is for returns taking place until August 2040. Where these resonance lines intersect the string of virtual asteroids, one finds a resonant return leading to a close approach. In the figure the string has been traced by using 1 001 multiple solutions (circles), and we have added a denser sampling of solutions (solid curve, as in Fig. 2) in the region near the 2027 closest approach. Note that for this argument it is essential to consider a continuous one-dimensional array of orbits, since none of the finite number of multiple solutions actually computed will be exactly resonant. But, if two consecutive virtual asteroids have periods straddling the resonant period, by continuity there is a nearby solution which is exactly resonant, and is compatible with the observations.

The other argument to be taken into account is the so called Minimum Orbital Intersection Distance (MOID), the minimum distance between the two osculating ellipses representing the orbit of the Earth and of the asteroid. Even if the asteroid were exactly on time at the rendezvous with the Earth, the unperturbed close approach distance cannot be less than the MOID. On the contrary, by the same continuity argument, if there is a virtual asteroid arriving late and one arriving early at the rendezvous, then there is a nearby solution arriving exactly on time, thus with a close approach distance as small as the MOID. If the MOID were to remain small forever a resonant return after h years, with a close approach distance as small as the MOID, would always be possible, since every real number P is approximated arbitrarily well by a rational number h/k. In fact, the MOID evolves with time, as a result of both short periodic and secular perturbations; the case of 1999 AN_{10} is special in that not only the MOID is very small (currently 0.0007 AU), but, according to secular perturbation theory [6, 7], it will remain small for about 600 years [14].

It is interesting to note that the mechanism of resonant returns is the primary means of delivering asteroids to the Earth's surface. This is somewhat obvious if one considers that during the period in which the node of the asteroid is crossing the Earth's orbit (over long time scales) there are generally numerous close encounters. It is unlikely that the very first encounter from the series of encounters will lead to an impact, hence, if a collision does occur, it will generally be a subsequent encounter.

3.3 Non-resonant returns

In some cases, close approaches can also occur far from the minimum distance point. This is because some near-Earth asteroids have a low distance from the orbit of the Earth at both the ascending and the descending node. This is the case for 1999 AN_{10}: the ascending node is only 0.00025 AU closer to the Sun than a point where the Earth is in early August; the descending node is 0.0048 AU inward from a position of the Earth in early February.

In such cases there is the possibility of a *non-resonant* return. This can occur if the Earth completes $h+1/2$ revolutions while the asteroid completes $k+1/2$ revolutions, so that they are both at the descending node at the same time. Taking into account the eccentricities of both orbits, the time required to go from the ascending node to the descending node is t_E for the Earth (not exactly half a year), and t_A for the asteroid (significantly different from half a period) [8]. Again allowing for the timing of the 2027 encounter, the condition to be satisfied for an encounter at the descending node is

$$h + t_E + \Delta t = k\,P + t_A\ .$$

If we take the conditions that both the Earth and the asteroid are at the appropriate node, and that the distance is zero at both nodes, we have 4 conditions on the 5 variables (a, e, ω, u_1, u_2), where u_1, u_2 are the eccentric anomalies at the nodes, and we can explicitly

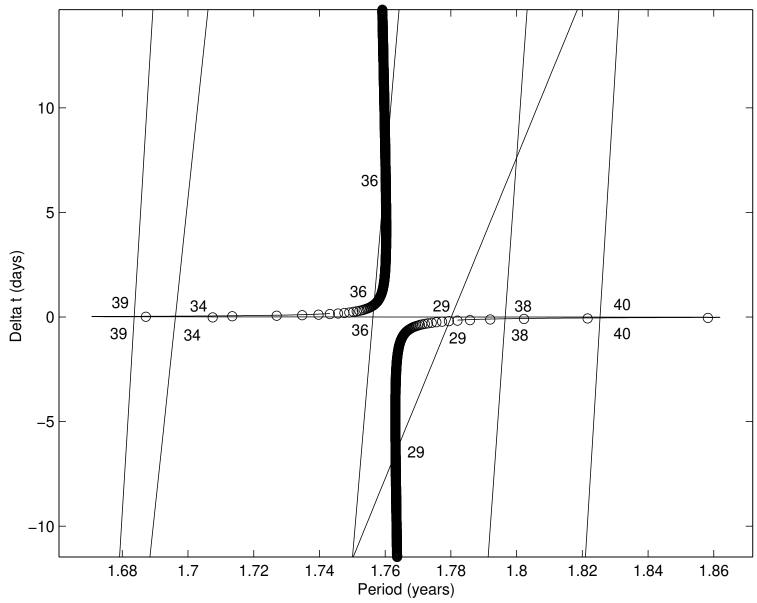

Figure 4: Non-resonant returns of 1999 AN_{10} at the descending node after the 2027 encounter, taking place until February 2040. The non-resonant returns are the intersections between the slanted lines and the string of multiple solutions, labelled with the year (after 2000) in which the return takes place.

compute t_A as a function of a. Thus the above conditions define a curve in the $(P, \Delta t)$ plane, as in the resonant case. Figure 4 shows all the possible non-resonant returns after the 2027 encounter with $h \leq 12$, that is taking place until February 2040. Again the intersections exist because the virtual asteroids are embedded in a continuous string. By explicitly listing the non-resonant returns (with the global method described in the next section) we find that the approximations used in this computation result in errors too small to be clearly visible at the scale of the figure.

Some asteroids, however, have a low inclination, and thus can have close approaches far from the nodes. The non-resonant returns of these low inclination cases are not easy to describe in theory, e.g., an analytic approach such as that of Fig. 4 would be difficult, but the fundamental mechanisms are the same.

If an inclined asteroid orbit has one node close to the orbit of the Earth and one close to the orbit of another planet, there can even be non-resonant returns to a different planet. As an example, the asteroids (5693) and 1997 NC_1 have complicated sequences of encounters with Earth and Venus, including both resonant returns to the same planet and non-resonant exchanges.

3.4 Secondary returns

Each of the close approaches resulting from either a resonant or a non-resonant return can be analyzed for its own resonant and non-resonant returns; these returns can spawn more returns, and so on. If the MOID remains small for a long time, as it is the case for very low inclination orbits (e.g., (4179) Toutatis [21]) and for high inclination, slowly precessing orbits (e.g., 1999 AN_{10} [14]), the sequence of these secondary returns leads to a quasi-fractal structure in the confidence region Z_X of initial conditions leading to encounters. (A true fractal structure could be obtained if the MOID was to remain small forever.)

In theory the cascade of successive returns could be described by a symbolic dynamics, as in other chaotic celestial mechanics problems [24]. In practice the proliferation of secondary returns is too complex to be analyzed using the theory of resonant returns. This implies that another method has to be used to predict the possible returns after some primary close approach has spawned a cascade of secondary returns that is too complicated to follow.

The stretching used in this paper is closely related to the dimensionless stretching used in the computations of the Lyapounov characteristic exponents: they differ only by a constant factor, that is the stretching at some initial epoch. In fact an orbit undergoing many close approaches is strongly chaotic, with a very short Lyapounov time (in which the orbit uncertainty grows by a factor exp(1)), typically a few years. The computation of an orbit over a time span of several decades during which close approaches occur goes beyond the predictability horizon; this forces the use of a probabilistic description of the subsequent motion. In the following section we shall discuss a way to compute the probability of an event occurring for one of these orbits, such as an impact.

The implications of return theory on the strategies to be used for a deflection initiative have also to be well understood. It is much easier to deflect an asteroid along track, provided a small change in semimajor axis can be introduced long before a threateningly close approach. But, if the orbit of an asteroid such as 1999 AN_{10} is deflected by an amount sufficient to avoid a specific return, but not enough to substantially change the MOID, other returns will form a different quasi-fractal structure in the post-maneuver confidence region. Thus, for the deflection to be safe, not only the currently threatening approach should be avoided, but also the other ones which can arise because of the deflection. A very fine understanding of all the possible resonant and non-resonant returns is required to be sure that the change in semimajor axis does not inject the asteroid into another threatening orbit.

4 Automatic monitoring

Our goal is to set up a procedure by which the orbits of all known Potentially Hazardous Asteroids (PHAs) would be monitored to ascertain if any one of them can possibly impact our planet before a given time t_f in the future; we generally use the year 2050. Automation is highly desirable, since the list of these objects is often increased by new discoveries, and the orbits have to be frequently updated as new observations become available. Our online information system NEODyS [20] automatically computes the orbits of all NEAs and detects the close approaches along the nominal solutions, but this is not enough.

This goal can be achieved if it is limited by stating that only the impacts with a probability above a given threshold need be detected. The reason of this limitation is clear if we think to the probability of an impact as a measure of the six-dimensional volume of the subset $K \subset Z_X$ containing the initial conditions leading to an impact. (The volume is the integral of a constant density function, and may need to be replaced by a probability integral of the probability density, but this does not change the argument.) Then, whatever the method used to select a finite sample of the confidence region, it has anyway a resolution limit. Once a minimum probability of impact has been fixed, it is possible to determine the number of virtual asteroids needed to achieve the corresponding resolution. The computing time needed to accurately propagate the orbits of all the virtual asteroids until t_f dictates the computing resources required; or alternatively, dictates the maximum refresh rate of the monitoring system, if it is not possible to recompute all the possible impacts each time an asteroid is observed.

Our currently operational procedure is not fully automated, it only handles the PHAs one by one, and is limited by the available computing power; at least in principle, however, it is a well defined algorithm, which we will describe in the rest of this section.

4.1 The catalog of virtual close approaches

The procedure starts with the computation of multiple solutions, as described in Sec. 3.1. If the spacing along the weakness line is $\Delta\sigma$ then the number of solutions X_i needed to span $\sigma \leq 3$ is $N = 6/\Delta\sigma$. Next the orbit is propagated from an epoch near the observations t_0 to the target time t_f for each initial condition X_i. To be prudent, each one of these orbits must be computed and propagated with top accuracy, i.e., with a dynamical model including all planets, the perturbations from the most massive asteroids, general relativistic corrections, and the Moon as a separate body. For all the N orbits, each close approach within the distance d is recorded with the geocentric coordinates of the closest approach point.

In many cases, several of the virtual asteroids X_i will experience a close approach to the Earth at around the same time t_k; this we call a *virtual shower.* A shower can be decomposed into separate *returns*, which are continuous strings of solutions having the same close approach; they are represented by sequences of n consecutive solutions $\{X_i,\ i = k, k+1, \ldots, k+(n-1)\}$. It is often the case that the shower at a given time contains several different returns [14, Table 1]; they generally can be interpreted as primary or secondary resonant/non-resonant returns. Each individual return follows a qualitatively different path to get to a rendezvous with the Earth at the same time.

An informative example of a shower is depicted in Fig. 5, which shows five returns from a single shower. The intersection of each virtual asteroid with the target plane is marked by a "+", and the points of each individual return are connected by straight lines. One of these returns, the one marked by two diamonds and a dotted line, actually allows for the possibility of collision. In this case it is clear that one must use caution in interpreting MTP results, since the dotted line clearly does not pass near the Earth, which is marked by the "$\oplus$". However, both of the diamonds are just slightly above the middle dashed line, as is the Earth; the true connection between the diamonds parallels the dashed lines and it does intersect the Earth.

We would like to note a few other points of interest in Fig. 5. The fact that the MTP stretching can change rapidly along the weak direction is quite clear from the spacing of points along the middle dashed line. Also observe that this shower has a single point return, or singleton, located above the other returns. Finally, the return closest to the bottom has a reversal in direction on the MTP. This is an example of an interrupted return, a phenomenon discussed in the next section.

4.2 The closest approach for each return

For each return we want to identify the closest possible approach in order to decide if an impact is possible. A first estimation of the minimum approach distance possible within

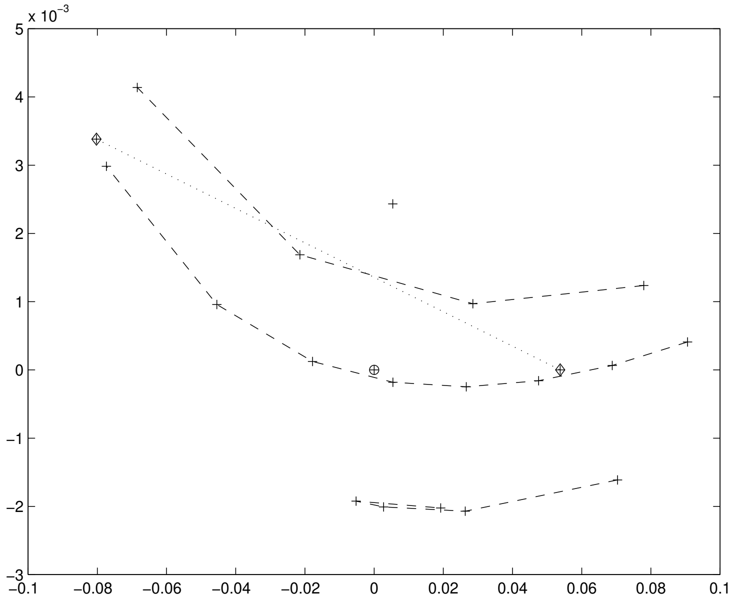

Figure 5: A single shower with five returns for the asteroid 1998 OX_4 in January 2046. The Earth is depicted by the "$\oplus$". The two diamonds that are connected by the dotted line actually correspond to a return with collision, emphasizing that care must be taken when interpolating between solutions. Note the axes are not to scale.

a given return is obtained by interpolating the value of the MOID of the nearby virtual asteroids. As discussed in Sec. 3.2, if the close approaches by consecutive virtual asteroids straddle the point of minimum distance, then there must be an orbit in the continuum of orbits that has a close approach at that distance. This distance is essentially the MOID; for small MOIDs, gravitational focusing results in a minimum distance somewhat lower than the MOID, as in Fig. 6. A correction to take into account gravitational focusing can be applied by a simple two-body formula.

The situation would be different if all the close approach points are on one side with respect to the minimum distance point; in this case we have an interrupted return, that is the value of the semimajor axis necessary to achieve the return is not compatible with the observations. As an example, in Fig. 2 we see that the value of the semimajor axis required to enter into the $h/k = 5/3$ resonance with the Earth, namely 1.405 AU, cannot be reached by 1999 AN_{10} after the 2027 encounter, no matter how close is the approach in that year (provided it is compatible with the observations). Thus, in August 2032 the return cannot result in an approach as close as the MOID because the asteroid is necessarily late for the rendezvous with the Earth; in fact the minimum possible approach distance is 0.03 AU. This can be seen from Fig. 7, which plots the points belonging to this particular return.

If the estimated minimum approach distance obtained through interpolation is very small, e.g., less than a few Earth radii, then it is not accurate enough. This is because the MOID changes rapidly as a result of the motion of the Earth with respect to the center of mass of the Earth-Moon system, and also because the MOID is just a 2-body approximation. Therefore in such cases we employ a more accurate iterative method to

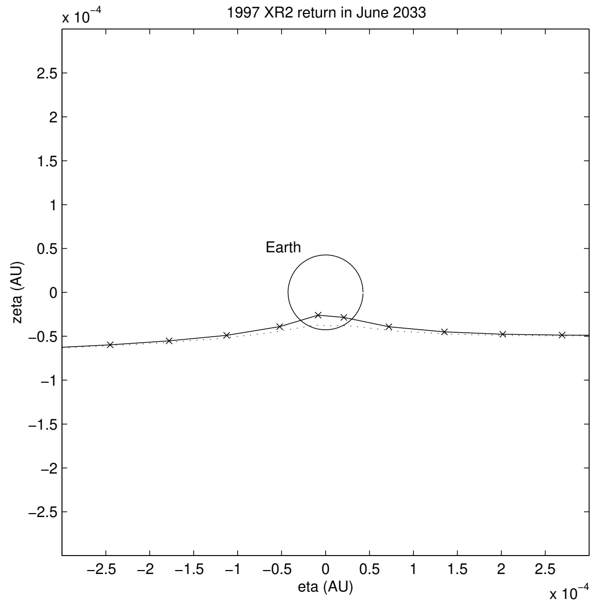

Figure 6: An example of possible impact. The effect of gravitational focusing is quite evident. (Please note that the multiple solutions used here are not really compatible with the observations; this is a hypothetical case, obtained by allowing for residuals larger than the acceptability threshold.)

calculate the minimum possible distance.

To this end we take the solution X_j that has the closest approach among the ones of the return and apply corrections to it to push the approach towards a closer one. The method is a variant of Newton's method, also called *differential corrections* in the context of orbit determination. It is similar, but not identical, to one method used in [18]. First we find the closest approach point on the MTP by a linear approximation, that is we find the point Y_{min} on the long axis of the MTP confidence ellipse which is closest to the Earth. The correction we would like to apply on the MTP is $\Delta Y = Y_{min} - F(X_j)$. Then we need to identify a change ΔX in the orbital elements (with respect to X_j), such that

$$DF\, \Delta X = \Delta Y\ .$$

A solution for ΔX can be found with a comparatively simple algorithm [15]; then we compute the orbit with initial conditions $X_j + \Delta X$ until a time $\simeq t^*$ and find the close approach; it defines a new MTP, on which we repeat the same procedure. The direction of the correction ΔY is recomputed at each step, thus the curvature introduced by gravitational focusing (see Fig. 6) is accounted for. The iterative scheme can be terminated when the distance from the Earth is not decreasing any more. In this way we have found the closest approaching orbit: note that it is in general significantly closer than the closest approach found among the finite set of virtual asteroids in the same return. If the minimum possible distance is less than one Earth radius, we have found a *virtual impactor*, that is one representative of the small subset of the confidence region for which collision occurs.

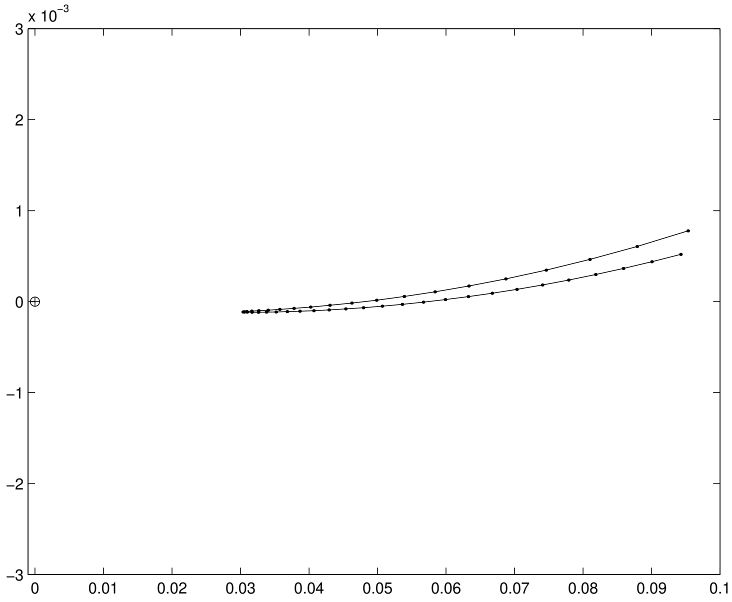

Figure 7: Interrupted return of 1999AN10 in 2032. The Earth is depicted by the "$\oplus$" at the origin. Note the axes are not to scale.

4.3 Estimation of the impact probability

If a virtual impactor has been found along the axis of the target plane confidence region, it is possible to estimate the probability of an impact with a very simple algorithm, provided that the confidence region is much narrower than the diameter of the Earth. In such cases a simple one-dimensional estimate is applicable, that is, only the probability density along the axis matters. The length of the interval in the σ-space corresponding to an impact is simply $d\sigma = 2\, R_\oplus / \Gamma_{MTP}$, where $R_\oplus$ is the radius of the Earth. (If the MOID is a significant fraction of $R_\oplus$ there is a correction for the chord length.) Then, if this length $d\sigma$ is small, the probability of the event occurring in this interval is just $P_d(\sigma)\, d\sigma$, with $P_d(\sigma)$ the assumed probability density. Thus the probability of an Earth impact is

$$P_{R_\oplus} = \frac{2\, R_\oplus}{\Gamma_{MTP}}\; P_d(\sigma) \ ,$$

where the MTP stretching Γ_{MTP} is computed from the partial derivatives of F at the MTP intercept of the virtual impactor, thus gravitational focusing is accounted for. When the width of the target plane confidence region is not negligible with respect to $R_\oplus$, it has to be accounted for in a fully two-dimensional probability computation. This requires the computation of a probability integral on the MTP, but even in this case, simple approximations can give the right order of magnitude.

For a uniform density on the span $|\sigma| \leq 3$, $P_d(\sigma) = 1/6$; for a gaussian univariate distribution, which is the marginal density derived on the one-dimensional σ-space from the six-dimensional gaussian distribution, we have $P_d(\sigma) \leq 0.4$, with maximum at the nominal solution. In practice, the real error distribution for asteroid astrometry is not uniform, yet it has much more tail than the gaussian, thus the truth is somewhat in between the two.

We can now give an approximate, but very useful, formula linking the probability of an impact and the density of the virtual asteroids along their string. Since two points on the MTP of some approach cannot be more distant than $2\,d = 0.2$ AU, then the returns with a stretching

$$\Gamma_{MTP} > \frac{2\,d}{\Delta\sigma} = \frac{0.2 \text{ AU}}{\Delta\sigma}$$

will contain no more than one point. For these singletons neither the MOID interpolation nor Newton's method are very reliable procedures, but in these cases we can give an upper bound for the corresponding probability of an impact

$$P_{R_\oplus} \leq \frac{2\,R_\oplus\ \Delta\sigma\ MaxP_d(\sigma)}{2\,d} \simeq 0.0043 \times MaxP_d(\sigma) \times \Delta\sigma\ .$$

That is $P_{R_\oplus} \leq 1.7 \times 10^{-4}\ \Delta\sigma$ for a gaussian distribution, and $P_{R_\oplus} \leq 7 \times 10^{-5}\ \Delta\sigma$ for a uniform distribution. As an example, the value we are currently using is $\Delta\sigma = 0.005$, implying the number of virtual asteroids is $N = 1201$, and the impacts are reliably detected (the returns being better than singletons) only if the probability is more than about 4×10^{-7}. For lower probability collisions, the encounter may be found as a singleton, but only with a likelihood given by the ratio of the true probability to the computed upper bound. Note that these computations are for an impact occurring with zero MOID and negligible width of the confidence region; grazing impacts can be detected at significantly lower probability levels.

Since the impact probability in this approximation is inversely proportional to the stretching, and therefore proportional to $\Delta\sigma$, the computational cost is inversely proportional to the probability limit. In practice, a modern personal computer is enough to monitor the impact possibilities down to a probability of one in a million, with a few hours of CPU time per asteroid (mostly consumed in the propagation of the virtual asteroid orbits).

4.4 Virtual impactor examples

Currently, the only asteroid with known collision solutions is 1998 OX_4. Table 1 provides details on the four 1998 OX_4 collision solutions that we have identified. The listed values of $P_{R_\oplus}$ assume the uniform probability density described above, i.e., $P_d(\sigma) = 1/6$. Unfortunately this very faint object was only observable for nine days and is now lost. In a recent paper [15] we have proposed a method of ensuring that the real 1998 OX_4 is not near the impacting solutions through a campaign of negative observations. This idea relies on the fact that, although the orbit of the asteroid is very poorly determined, the orbit of a "virtual impactor" is very precisely known because the impact itself severely constrains the orbit. Hence all that is required is to point a telescope towards the position of the hypothetical impactor at some favorable observing opportunity and see if it is there or not.

We also present two interesting examples of objects for which collisions solutions did exist, but were later ruled out by the inclusion of precovery observations. The previously discussed asteroid 1999 AN_{10} was recovered in late May 1999, and continued to be observed through August. It is interesting to briefly describe its situation as of July 9, 1999, when there were 118 observations published by the Minor Planet Center over the period 13 January to 8 July 1999. With the orbit from this dataset we have identified two collision solutions (Table 2). Observations of this asteroid at its 1955 apparition have been located

Table 1: 1998 OX_4 POTENTIAL COLLISION SOLUTIONS

Year of Impact	2014	2038	2044	2046
Date (UT)	Jan 20.5	Jan 20.2	Jan 20.7	Jan 20.3
σ	2.422	0.746	0.760	0.779
Γ_{MTP} (AU/σ)	397	95	47	18
$P_{R_\oplus}$	0.3×10^{-7}	1×10^{-7}	2×10^{-7}	2×10^{-7}

[17], and the greatly refined orbit solution thus obtained does not allow collision possibilities for at least 150 years in the future.

Table 2: 1999 AN_{10} POTENTIAL COLLISION SOLUTIONS

Year of Impact	2044	2069
Date (UT)	Aug 6.7	Aug 6.9
σ	2.85	0.34
Γ_{MTP} (AU/σ)	1.6	320
$P_{R_\oplus}$	9×10^{-6}	3×10^{-8}

The other example is somewhat similar to 1999 AN_{10}, except that the potential collision solutions were in fact unknown until long after the precovery observations were already available. The asteroid 1997 XF_{11} received considerable public attention in March of 1998 due to a supposed possibility of collision in October 2028. Although the 2028 collision was in fact not compatible with the observations available at the time, precovery observations dating to 1990 were found almost immediately, and the resulting multi-opposition orbit enabled orbit computers to rule out any possible collision for the foreseeable future. (See Ref. [12] for historical details.) The collision solutions depicted in Table 3 result from the 107 day arc of observations from 12 December 1997 to 23 March 1998. The table shows only solutions with impact before 2050 and probability of impact greater than 10^{-7}.

Table 3: 1997 XF_{11} POTENTIAL COLLISION SOLUTIONS

Year of Impact	2039	2040	2041	2043	2047	2048
Date (UT)	Oct 27.5	Oct 26.7	Oct 27.0	Oct 27.5	Oct 27.6	Oct 26.8
σ	0.33	2.21	0.16	0.12	2.22	0.26
Γ_{MTP} (AU/σ)	21	0.33	15	24	34	30
$P_{R_\oplus}$	5×10^{-7}	4.5×10^{-5}	1×10^{-6}	6×10^{-7}	3×10^{-7}	4×10^{-7}

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