OPTIMIZATION OF SPACE SURVEILLANCE RESOURCES BY INNOVATIVE PRELIMINARY ORBIT METHODS

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ABSTRACT

The number and performance of the sensors to be used for a survey is a function of the minimum number of observations required to determine an orbit. This is critical for the definition of the sensor network and the observation planning.

Our goal is to obtain an orbit with a smaller number of observations with respect to the classical methods, such as Gauss/Laplace. In the context of space debris surveys, the goal is a full 6-elements orbit from just 2 tracklets, which could be obtained with only 2 exposures. The information contained in a tracklet can be summarized in a 4-dimensional vector called attributable, thus two tracklets are enough for the orbit determination problem to be over-determined.

We have proposed an algorithm based upon the integrals of the 2-body problem. We outline the equations and the solution methods which are used in our implementation. We report on the results of a validation test, based upon the processing of one year of data from ESA Optical Ground Station. We conclude that the method is very effective and can be used to find correlations between tracklets, to be confirmed with additional correlations, thus providing a catalog of full 6-elements orbits.

Key words: orbit determination; correlation; integrals of motion.

1. INTRODUCTION

To convert a set of astrometric observations into a catalog of orbits of satellites and space debris we have to solve the problem of *correlation*, that is to find which data belong to the same physical object. This is strictly analogous to the problem called *identification* in the context of asteroid surveys [6]. The problem could be difficult because of the nonlinear nature of the underlying dynamical system and of the high computational complexity. Thus there is a trade-off between using a more computationally aggressive correlation/identification method and tightening the requirements on the amount and distribution in time of the observational data.

When using observations already stored, there is no choice but trying to extract as much results, that is correlations and good orbits, as it is possible from the available data: this is one reason why there has been significant progress in the definition and testing of algorithms for asteroid identification, and the technological transfer of these to the correlation problem for debris is possible. On the contrary, when designing a new survey, it is necessary to include the requirements on the correlation and orbit determination procedure. In this case, the trade-off between using more aggressive correlation algorithms and building an observing network with higher performance clearly turns in favor of the first option.

This is the reason why we have been trying to convert our expertise on asteroid identification and orbit determination into the corresponding capabilities for correlation of space debris. We have asked for the opportunity to use an existing data set of observations from ESA Optical Ground Station to validate our algorithms, taking into account that of course the observation scheduling cannot have been optimal for our methods. We thank the University of Bern (in particular T. Schildknecht) and ESA for providing these data and for supporting our research.

2. OBSERVATIONS AND ATTRIBUTABLES

How are the observations of debris obtained and what is their information content?

In the special case of a survey of the geosynchronous region the observations can be taken by stopping the telescope motor, thus in a reference frame body-fixed with the Earth. Then the stars appear as long trails, the nearly geostationary objects as very short ones or even points, the other debris as medium to long trails. The ends of all trails are measured: the ones of the stars are used for astrometric reduction, the ones for the moving objects are converted into two positions taken at the beginning and the end of the exposure, forming a *tracklet*. This technique has been known for a long time: for an example, see [2, Fig. 1, 2].

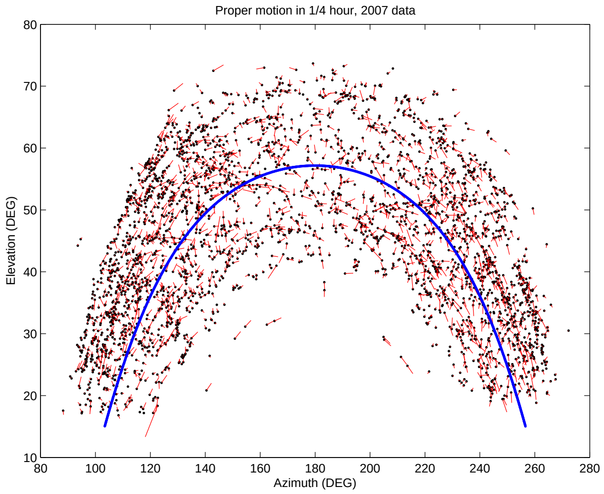

Figure 1. Validation dataset: 2007 tracklets from ESA OGS, pre-processed by University of Bern and converted to DES. Angles are in a body-fixed altazimutal reference. Angular velocity is represented by the motion in 1/4 hour. The blue line is the geostationary line (where GEO with $e = I = 0$ are found).

The data we have used have been obtained by the ESA Optical Ground Station (OGS) at Teide Observatory (Canary Islands) in the year 2007. They were collected in a survey targeted at the geosynchronous belt, although of course objects in different orbits were incidentally imaged; the region being surveyed was a belt above and below the *geosynchronous line*, where exactly circular, equatorial and geosynchronous orbits could be seen from the OGS location. Fig. 1 shows a global view of this data set in a body-fixed reference frame.

After astrometric reduction performed by University of Bern, the data have been converted to a Data Exchange Standard (DES), defined by us in collaboration with the next generation asteroid surveys Pan-STARRS and LSST, thus it is expected to become a *de facto* standard for asteroid observations [10].

When these data are converted into an *inertial sidereal frame*, defined by the star catalog used, the information content of the tracklet is equivalent to an *attributable* [8], that is a 4-dimensional vector

$$A = (\alpha, \delta, \dot{\alpha}, \dot{\delta})$$

with two angular coordinates (e.g., right ascension and declination) and their time derivatives, all referred to the average time $\bar{t}$ of the exposure. The topocentric distance ρ and its time derivative $\dot{\rho}$ remain completely unknown.

Note that even $\dot{\alpha}, \dot{\delta}$ are well determined: in the inertial frame the Geosynchronous Earth Orbits (GEO) move by 900 arcsec/min and we can have $\simeq 10 \times 2$ bits of additional information with respect to the position only.

A set of observations giving an attributable is not enough to compute an orbit, unless some restrictive hypothesis is used. In fact with these data we have a 2-dimensional manifold of possible orbits that give exactly the same attributable at a given time. Thus to complete an orbit we need either to assume 2 coordinates, or to set 2 constraints, e.g., assuming a *circular orbit*, that is a good approximation for geostationary objects but not for geosynchronous ones, which may have a significant eccentricity.

To define an orbit given the attributable A we need to find the values of the topocentric *range* ρ and *range-rate* $\dot{\rho}$, that, together with the attributable, give us a set of *attributable orbital elements*

$$X = [\alpha, \delta, \dot{\alpha}, \dot{\delta}, \rho, \dot{\rho}] = [A, \rho, \dot{\rho}]$$

at a time $\tilde{t}$, computed from $\bar{t}$ taking into account the light-time correction: $\tilde{t} = \bar{t} - \rho/c$. The Cartesian geocentric position and velocity $(\mathbf{r}, \dot{\mathbf{r}})$ can be obtained, given the observer geocentric position $\mathbf{q}$ at time $\bar{t}$, by using the unit vector $\hat{\boldsymbol{\rho}}(A)$ in the direction of the observation:

$$\mathbf{r} = \mathbf{q} + \rho\,\hat{\boldsymbol{\rho}}(A) \;\;,\;\; \dot{\mathbf{r}} = \dot{\mathbf{q}} + \dot{\rho}\,\hat{\boldsymbol{\rho}}(A) + \rho\,\frac{d\hat{\boldsymbol{\rho}}(A)}{dt}\;.$$

Thus the question is the correlation problem: given two attributables A_1, A_2 at different times $\bar{t}_1, \bar{t}_2$, can they belong to the same orbiting object? And if this is the case, how can we find an orbit fitting both data sets? Note we are not assuming the time difference $\bar{t}_2 - \bar{t}_1$ is small: it could even be several days, that is several orbital periods for a GEO.

3. THE KEPLERIAN INTEGRALS METHOD

We assume that the orbit between $\tilde{t}_1$ and $\tilde{t}_2$ is well approximated by a Keplerian 2-body orbit, with constant angular momentum vector $\boldsymbol{c}$ and energy $\mathcal{E}$. Their expressions for a given attributable A are

$$\boldsymbol{c}(\rho, \dot{\rho}) = \mathbf{r} \times \dot{\mathbf{r}} = \mathbf{D}\dot{\rho} + \mathbf{E}\rho^2 + \mathbf{F}\rho + \mathbf{G}$$

where the vectors $\mathbf{D}, \mathbf{E}, \mathbf{F}, \mathbf{G}$ are obtained from vector products of the known vectors $\mathbf{q}, \hat{\boldsymbol{\rho}}, \dot{\mathbf{q}}, d\hat{\boldsymbol{\rho}}/dt$, and

$$2\mathcal{E}(\rho, \dot{\rho}) = \dot{\rho}^2 + c_1\dot{\rho} + c_2\rho^2 + c_3\rho + c_4 - \frac{2\,\mu_\oplus}{\sqrt{\rho^2 + c_5\rho + c_0}}$$

where $\mu_\oplus$ is the mass of the Earth times the gravitational constant, and the coefficients $c_j, j = 0, 5$ are obtained from scalar products of the known vectors $\mathbf{q}, \hat{\boldsymbol{\rho}}, \dot{\mathbf{q}}, d\hat{\boldsymbol{\rho}}/dt$. If we assume that the values $\mathcal{E}_j, \boldsymbol{c}_j$ at time $\tilde{t}_j$ are computed from A_j with unknowns $\rho_j, \dot{\rho}_j$, then from $\boldsymbol{c}_1 = \boldsymbol{c}_2$ we get

$$\boldsymbol{c}_1 = \boldsymbol{c}_2 \Longleftrightarrow \mathbf{D}_1\dot{\rho}_1 - \mathbf{D}_2\dot{\rho}_2 = \mathbf{J}(\rho_1, \rho_2)$$

where $\mathbf{J}(\rho_1, \rho_2)$ is quadratic in the unknown ranges. By scalar product with $\mathbf{D}_1 \times \mathbf{D}_2$ we eliminate $\dot{\rho}_1, \dot{\rho}_2$ and obtain the scalar algebraic equation of degree 2:

$$\mathbf{D}_1 \times \mathbf{D}_2 \cdot \mathbf{J}(\rho_1, \rho_2) = q(\rho_1, \rho_2) = 0\;.$$

Geometrically, this equation defines a conic section in the (ρ_1, ρ_2) plane, in most cases a hyperbola.

By the formula giving $\dot\rho_1, \dot\rho_2$ as a function of ρ_1, ρ_2 derived from the angular momentum equations, the energies $\mathcal{E}_1, \mathcal{E}_2$ can be considered as functions of ρ_1, ρ_2 only. If we assume that the energy and angular momentum at $\tilde t_1$ and $\tilde t_2$ are the same, then we get the system of 2 equations in 2 unknowns:

$$\left\{ \begin{array}{rcl} \mathcal{E}_1(\rho_1, \rho_2) - \mathcal{E}_2(\rho_1, \rho_2) & = & 0 \\ q(\rho_1, \rho_2) & = & 0 \end{array} \right. .$$

These equations were already present in [11] for Earth satellites: they proposed a Newton-Raphson method to solve the system, but this results into a loss of control on the number of alternate solutions. [5] have applied the same equations to the asteroid problem, and proposed a different approach to the solution of the system.

The energy equation is *algebraic, but not polynomial*, because there are denominators containing square roots. By squaring twice it is possible to obtain a polynomial equation $p(\rho_1, \rho_2) = 0$: the degree of this equation is 24. Thus the system

$$\left\{ \begin{array}{rcl} p(\rho_1, \rho_2) & = & 0 \\ q(\rho_1, \rho_2) & = & 0 \end{array} \right.$$

has exactly 48 solutions in the complex domain, counting them with multiplicity. Of course we are interested only in solutions with ρ_1, ρ_2 real and positive, moreover the squaring of the equations introduces spurious solutions. Nevertheless, we have found examples with up to 11 non-spurious solutions.

We need a global solution of the algebraic system of overall degree 48, providing at once all the possible couples (ρ_1, ρ_2). This is a classical problem of algebraic geometry, which can be solved with the *resultant method*: we can build an auxiliary Sylvester matrix, in this case 22×22, with coefficients polynomials in ρ_2, and its determinant, the resultant, is a polynomial of degree 48 in ρ_2 only. The values of ρ_2 appearing in the solutions of the polynomial system are the roots of the resultant [3].

The computation of the resultant is numerically unstable, because the coefficients have a wide range of orders of magnitude: we had to use quadruple precision. Once the resultant is available, there are methods to solve the univariate polynomial equations, providing at once all the complex roots with rigorous error bounds [1]. Given all the roots which could be real, we solve for the other variable ρ_1, select the positive couples (ρ_1, ρ_2) and remove the spurious ones due to squaring. If the number of remaining solutions is 0, we can assume the tracklets are not correlated.

The procedure above is somewhat slow, because extended precision is emulated in software. Moreover, for M tracklets we need to run the algorithm $M^2/2$ times. However, this computation is trivially parallelizable: for $\simeq$ 3172 tracklets we used 6 cores of standard CPUs to complete the computation in less than 2 hours. Nevertheless, we are working to improve the computational speed, with the goal of proposing a final algorithm which is substantially faster, therefore suitable for much large data sets.

4. PRELIMINARY ORBITS

Once the distances (ρ_1, ρ_2) are available, $(\dot\rho_1, \dot\rho_2)$ are computed from the angular momentum equations and the values of attributable elements can be obtained for the epochs $\tilde t_1$ and $\tilde t_2$, and they can be converted into the usual Keplerian elements:

$$(a_j, e_j, I_j, \Omega_j, \omega_j, \ell_j)\ ,\ j = 1, 2$$

where $\ell_j = n_j\,(\tilde t_j - t_{0j})$ are the mean anomalies. However, the first four Keplerian elements a_j, e_j, I_j, Ω_j are functions of the 2-body energy and angular momentum vectors $\mathcal{E}_j, \boldsymbol{c}_j$, and these are the same for $j = 1, 2$. Thus the result is

$$V = (a, e, I, \Omega, \omega_1, \ell_1, \omega_2, \ell_2)$$

and there are *compatibility conditions* to be satisfied if the two attributables belong to the same object:

$$\omega_1 = \omega_2\ \ ,\ \ \ell_1 = \ell_2 + n(\bar t_1 - \bar t_2)\ .$$

We cannot check the exact equality in the formulae above, because of various error sources, including the astrometric uncertainty of the attributable, especially the 1/1000 fractional error in $\dot\alpha, \dot\delta$, and the perturbations on the Keplerian integrals mostly due to the second harmonic of the Earth's gravity field and the lunisolar tidal attraction.

The multiple orbits obtained from the solutions of the algebraic problem are just *preliminary orbits*, solution of a 2-body approximation (as in the classical methods of Laplace and Gauss).

In our method the preliminary orbits are endowed with a *covariance matrix* which is obtained by propagating the available 8×8 covariance matrix of (A_1, A_2) to the covariance matrix of the 8-vector V. With another simple transformation, this provides a 2×2 covariance matrix for the compatibility conditions on the variables ω_j, ℓ_j. Thus we can compute a χ^2 value for the discrepancy of the compatibility conditions, and use this as quality control parameter, that is we accept the output as preliminary orbit if and only if $\chi \leq \chi_{max}$; in our tests we have used values of χ_{max} between 5 and 10.

We have validated this algorithm by running it on the 3172 tracklets from the 2007 OGS observations. We have limited the time interval to $|\bar t_2 - \bar t_1| \leq 10$ days, to avoid the excessive accumulation of perturbations making the 2-body preliminary orbit a poor approximation; we do not yet know what is the maximum usable time span.

The accepted preliminary orbits are used as starting guess for differential corrections; if they are convergent, we accept the 2-tracklet correlation with the orbit from the least squares fit.

5. CORRELATION CONFIRMATION

With our method we have found 363 correlations of 2 tracklets, with 378 accepted orbits. However, these need to be confirmed, because a least squares solution with 8 equations in 6 unknowns is weak, and the fit could be good even for a false correlation. Not to speak of the 15 cases with two significantly different orbits, where we do not know how to choose among the 2.

Correlation confirmation is best obtained by looking for a third tracklet which can also be correlated to the other 2; this process is called *attribution* [6, 8]. From the available 2-tracklet orbits with covariance we predict the attributable A_P at the time $\bar{t}_3$ of the third tracklet, and compare with the attributable A_3 computed from the third tracklet. Since both A_p and A_3 come with a covariance matrix, we can compute the χ^2 of the difference and use it as a test. For the attributions passing this test we proceed to the differential corrections [9].

The procedure is *recursive*, that is we can use the 3-tracklet orbit to search for attribution of a fourth tracklet, and so on. This generates a very large number of many-tracklet orbits, but there are many duplications, corresponding to adding the tracklets in a different order.

By *correlation management* we mean a procedure to remove duplicates (e.g., $A = B = C$ and $A = C = B$) and inferior correlations (e.g., $A = B = C$ is superior to both $A = B$ and to $C = D$, thus both are removed). The output catalog after this process is called normalized. In the process, we may try to merge two correlations with some tracklets in common, by computing a common orbit fitting all the data; this process is found to succeed sometimes, but not always.

The output of our test with the 2007 OGS data, after correlation management, included 206 correlations (with 220 orbits). Of these, 112 were not confirmed, that is limited to two tracklets, therefore the orbit is weakly constrained and the correlation itself is not fully reliable.

T	2	3	4	5	6	7	8	9	10	12
C	112	40	29	10	3	5	3	1	1	1

Table 1. C is the number of correlations found with T tracklets.

Out of 3172 input tracklets, 464 have been correlated, 2708 left uncorrelated. The 2007 observations were not scheduled to allow for orbit determination of all the objects.

Of course we have no way to know how many should have been correlated, that is how many physically distinct objects are there: in particular, objects re-observed at intervals longer than 10 days have escaped correlation.

6. POPULATION ASSESSMENT

Do the results of the previous section correspond to a reasonable population model?

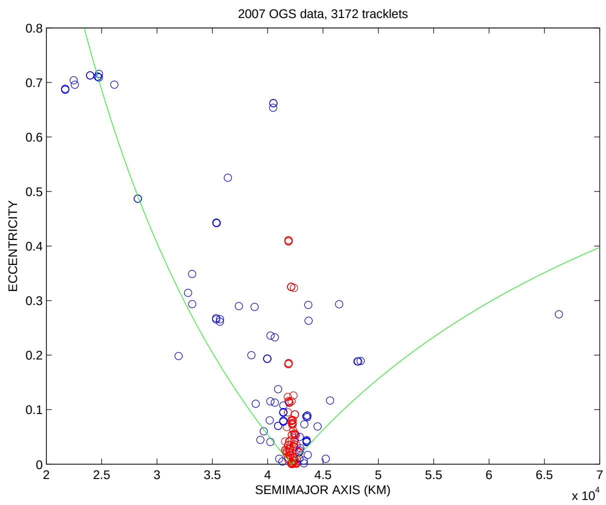

Figure 2. Orbits determined with our method projected on the (a, e) plane: the green lines indicate apocenter (on the left) and pericenter (on the right) at the geostationary altitude. The GTO are on the apocenter line.

The population which is observed by surveying around the geostationary line (see Fig. 1) contains geostationary objects, with low values of e and I and geosynchronous (or almost) objects which could have a significant e and I, including some very high values which could occur for large Area/Mass (A/M) as a result of radiation pressure [12].

Some of the observed tracklets also belong to objects in orbits with semimajor axis very different from the one of the GEO: as an example there could be Geosynchronous Transfer Orbits (GTO). The latter can be easily identified because they should have an apocenter at the geosynchronous altitude (or almost).

The orbits in the (a, e) plane (Fig. 2) show a concentration of GEO, including some with high e.

The orbit with pericenter well above the geosynchronous height is a non confirmed correlation which is likely to be false. Indeed, one of the two tracklets can be correlated with another one, giving a GTO orbit. This is a good example of the fact that uncorrelated correlations should not be trusted, although many of them could be true.

Fig. 3 shows that there are two groups of orbits with $e \simeq 0.32$ and $e \simeq 0.41$, and they also correspond to a quite large inclination: $I \simeq 17^\circ.5$ and $I \simeq 10^\circ$. These

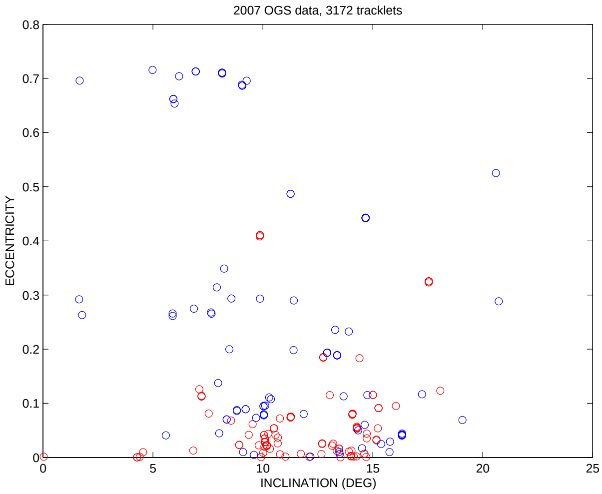

Figure 3. Orbits from our method projected on the (I, e) plane: the red dots are nearly geosynchronous orbits, with a within 700 km from the geosynchronous value. The few GEO orbits with high e and I should have high area/mass ratio.

values could be reached in few years by originally geostationary debris provided they have A/M of the order of 20 m^2/kg [12, Fig. 3, 6]. The two groups of orbits actually correspond just to two objects, because the correlation was not achieved. For the values of A/M cited above, the radiation pressure perturbation is much larger than the ones due to Earth's spherical harmonics, the Moon and the Sun. Thus a least squares fit over a time span of many days must necessarily fail, unless we have a radiation pressure model. For now we have only order of magnitude guesses for the radiation pressure model parameters, including A/M (and other parameters, since the shapes are certainly not spherical). If we had a much larger data set of observations we could estimate the values of these parameters and presumably correlate all the observations of these two objects.

Fig. 2 clearly shows a number of GTO orbits: from Fig. 3 we can check that they have moderate inclinations.

Fig. 3 also shows an apparent lack of really geostationary orbits, with low e and I: actually there is only one orbit with $e < 0.01$ and $I < 2°$. This is due to the fact that the survey conducted by the OGS in 2007 had the purpose of discovering new objects, and the geostationary objects are mostly active satellites, whose orbits and ephemerides are known. Thus the fields of view were on purpose avoiding the geostationary line of Fig. 1.

Fig. 4 and 5 show the distribution of eccentricity/inclination versus intrinsic luminosity of the objects, the latter described in the absolute magnitude scale. Unfortunately it is not easy to convert an absolute magnitude into a size, because of the wide range of albedo values and also because of irregular shapes. However, if we could assume albedo 0.1 and a spherical shape, we would get a diameter ranging between 10 m and $\simeq 30$ cm for

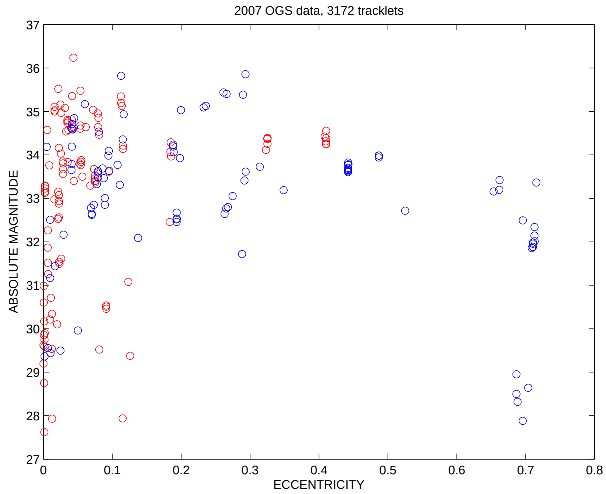

Figure 4. Orbits from our method projected on the (e, H) plane, where H is the absolute magnitude. $H = 28$ would correspond to about 10 m diameter, $H = 33$ to 1 m, if the shape was spherical and albedo was 0.1, like the one of the average asteroid. The largest objects should be satellites (lower left) and rocket stages (lower right).

the correlated objects. Thus the largest objects should be satellites (at low e) and rocket stages (near GTO), the smallest are certainly debris.

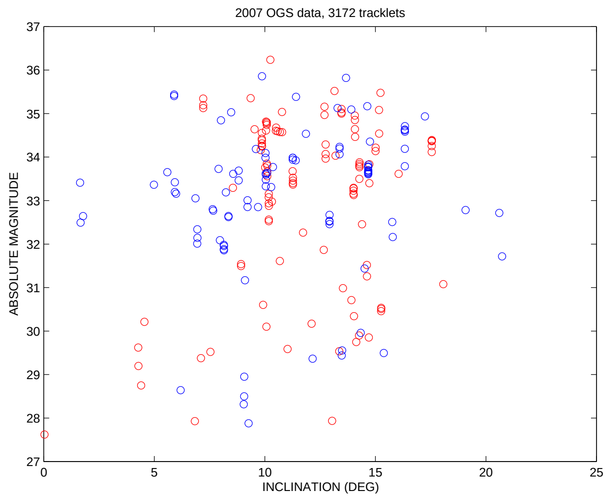

Figure 5. Orbits determined with our method projected on the (I, H) plane. There is a geosynchronous population with moderately high inclination, and a wide range of sizes.

The existence of objects with high e and also I was already well known, what is interesting is that some of these have a quite large cross section. To understand the dynamics of such objects is a challenge, which requires advanced models and a good data set of both astrometry and photometry.

7. CONCLUSIONS AND FUTURE WORK

1. We have developed and validated with one year of OGS data an innovative and more aggressive orbit determination method, allowing to use *just one tracklet per night* (for GEO, one exposure). The time span between two tracklets can be as long as 10 days. The orbits are complete, with 6 elements.

2. We find *hundreds of correlations*, even in a dataset which was taken without following a strategy optimized for orbit determination. The resulting orbits make sense, thus most of them should be right, although a few may be wrong (especially the ones not confirmed by correlating a third tracklet).

3. The resulting catalog of orbits is not yet in a 1-1 correspondence with real objects, because objects observed too far apart in time cannot be fit to a purely gravitational orbit. This phenomenon is especially relevant for large area/mass objects (with A/M $= 1$ m^2/kg, the radiation pressure is as important as the J_2 perturbation).

4. The scheduling of the next generation debris survey can be based on the assumption that just one tracklet, providing a good attributable, is required for each object and for each night. For a given telescope performance, the classical methods based upon three exposures per night require more telescopes. For a method requiring 2 tracklets per night, see the paper by Tommei et al. in these proceedings.

5. The same method can be applied to MEO/HEO orbits, provided the exposure (with sidereal tracking) is long enough to give the angular velocities $\dot{\alpha}, \dot{\delta}$ with good accuracy. By using 1 second exposure this is not the case, then a tracklet requires two exposures. Apart from this, there is nothing in our method specific for GEO.

6. The same method, or a conceptually similar one, can be applied with radar data, provided the radar measures the quantities $(\rho, \dot{\rho}, \alpha, \delta)$, a 4-dimensional *radar attributable*, with all components accurate enough. The equations are actually easier than for optical data [7, 4]. We have not tested this, also because we have not yet been supplied with validation data.

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