Proper elements for Earth-crossers

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The orbits of planet-crossing asteroids (and comets) can undergo close approaches and collisions with some major planet. This introduces a singularity in the N-body Hamiltonian, and the averaging of the equations of motion, traditionally used to compute secular perturbations, is not defined. Planet-crossing orbits have been extensively studied, by numerical integrations, and these studies have led to the conclusion that the secular resonances are the main dynamical phenomenon controlling their long term evolution. These resonances have been detected empirically as librations of some critical argument in the numerical integration output; however, to predict where in the phase space these secular resonances occur, a definition of proper frequencies for planet-crossing orbits is required, together with an explicit algorithm to compute them. In particular for the Earth-crossing region it is known that secular resonances with the perihelia and nodes of Mars and of the Earth can occur, but the theories available so far were not considering the perturbations by the terrestrial planets for the orbits which have node crossings with them.

We have introduced a generalized definition of averaged solution, which has a rigorous meaning also for planet-crossing orbits, applicable in the approximation neglecting the eccentricities and mutual inclinations of the planetary orbits, but without limitations on the eccentricities and inclinations of the asteroids/comets. This allows to define proper frequencies of precession of the perihelia and nodes, and also proper elements, for planet crossing orbits with arbitrary semimajor axis, eccentricity and inclination. A systematic exploration of the phase space by the corresponding semianalytic theory would allow to identify the secular resonance surfaces in the planet-crossing region. This needs to be followed by numerical integration tests to verify the accuracy and even applicability of this theory, which is expected to be extremely non uniform, depending upon the geometry of the node crossings and upon the proper frequencies themselves.